Answers for class prep quiz on section 3.5, Stewart's Calculus (8th ed.)

1. **Answer:** (a). Taking the derivative of both sides of $ye^{xy} = 7$, we get:

$$\begin{aligned}\frac{dy}{dx}e^{xy} + ye^{xy}\left(x\frac{dy}{dx} + y\right) &= 0\\ \frac{dy}{dx}\left(e^{xy} + xye^{xy}\right) &= -y^2e^{xy}\\ \frac{dy}{dx} &= \frac{-y^2e^{xy}}{e^{xy} + xye^{xy}}.\end{aligned}$$

2. **Answer:** (c). Taking the derivative of both sides of $x^3y - 2y^2x = -12$, we get:

$$\begin{aligned}3x^2y + x^3\frac{dy}{dx} - 4y\frac{dy}{dx}x - 2y^2 &= 0\\ \frac{dy}{dx}(x^3 - 4yx) &= 2y^2 - 3x^2y\\ \frac{dy}{dx} &= \frac{2y^2 - 3x^2y}{x^3 - 4yx}\end{aligned}$$

   Plugging in $x = 2$, $y = -1$, we get

$$\frac{dy}{dx} = \frac{2+12}{8+8} = \frac{7}{8},$$

   so by point-slope, the equation of the tangent line is

$$(y+1) = \frac{7}{8}(x-2).$$

3. **Answer:** (d). Since $y = \sin^{-1}(x)$ implies that $\sin y = x$, by differentiating both sides of $\sin y = x$, we get

$$\begin{aligned}\cos y\left(\frac{dy}{dx}\right) &= 1\\ \frac{dy}{dx} = \frac{1}{\cos y} &= \frac{1}{\sqrt{1-x^2}}\end{aligned}$$

   for $-\pi/2 \le y \le \pi/2$, the range of $y = \sin^{-1}(x)$.

4. **Answer:** (a). Since $y = \ln x$ implies that $e^y = x$, by differentiating both sides of $e^y = x$, we get

$$e^y\left(\frac{dy}{dx}\right) = 1$$
$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}.$$

Therefore, $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$.